A

NUMBER SETS

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- if V is the set of all vowels, then $V = \{\text{vowels}\} = \{a, e, i, o, u\}$
- if E is the set of all even numbers, then $E = \{\text{even numbers}\} = \{2, 4, 6, 8, 10, 12, \ldots\ldots\}$.

We use the symbol $\in$ to mean *is an element of* and $\notin$ to mean *is not an element of*.

So, for the set $E = \{2, 4, 6, 8, 10, 12, \ldots.\}$, we can say $6 \in E$ but $11 \notin E$.

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The set $\{\ \}$ or $\varnothing$ is called the **empty set** and contains no elements.

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For example:

- the set of vowels V has 5 elements. V is a finite set, and $n(V) = 5$
- the set of even numbers E is an infinite set.

SPECIAL NUMBER SETS

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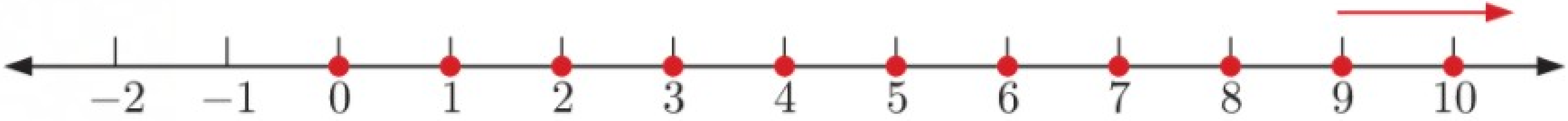
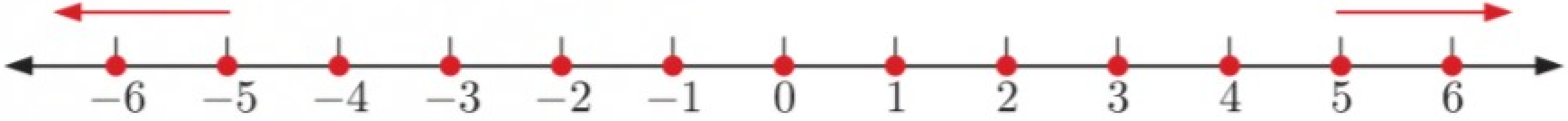
Following is a list of some special number sets you should be familiar with:

- $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7, ...\}$ is the set of all **natural** or **counting numbers**.



- $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, ...\}$ is the set of all **integers**.



- $\mathbb{Q}$ is the set of all **rational numbers**, or numbers which can be written in the form $\frac{p}{q}$ where p and q are integers, $q \neq 0$.

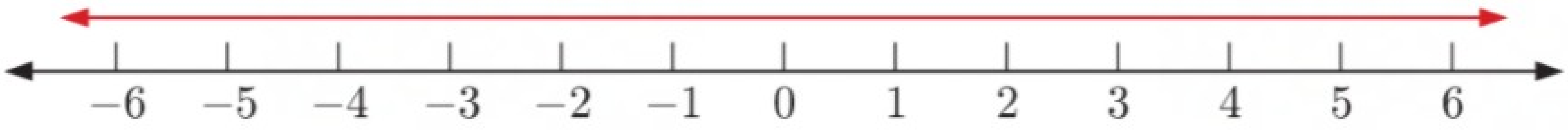
For example: $\frac{15}{4}$, $10\ (= \frac{10}{1})$, $0.5\ (= \frac{1}{2})$, and $-\frac{3}{8}$ are all rational numbers.

We cannot represent the rational numbers on a number line, because there are infinitely many of them, and in between them are **irrational numbers** which cannot be written in rational form.

For example:

- Radicals or surds such as $\sqrt{2}$ and $\sqrt{7}$ are irrational.
- $\pi \approx 3.141\,592\,65$ is an irrational number.
- Decimal numbers which neither terminate nor recur are irrational.

- $\mathbb{R}$ is the set of all **real numbers**, which are all numbers which can be placed on the number line.



$\mathbb{R}$ includes all rational and irrational numbers.

$\frac{2}{0}$ and $\sqrt{-2}$ are not real numbers because we cannot write them in decimal form or place them on a number line.

EXERCISE 2A

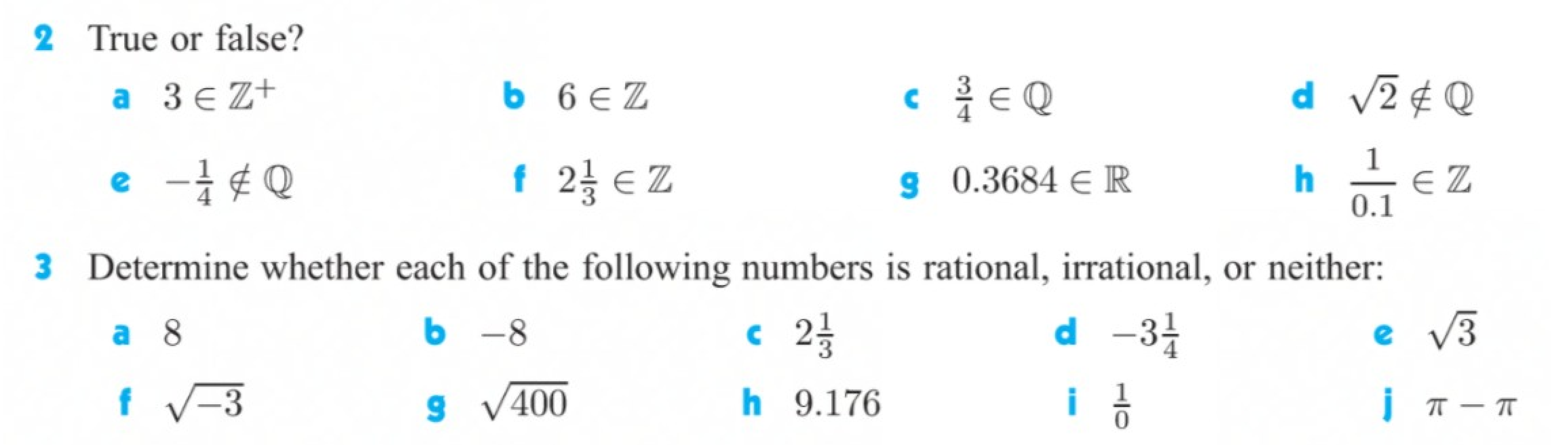
1 Write using set notation:

  a 8 is an element of set P.

  b k is not an element of set S.

  c 14 is not an element of the set of all odd numbers.

  d There are 9 elements in set Y.



2 True or false?

a $3 \in \mathbb{Z}^+$
b $6 \in \mathbb{Z}$
c $\frac{3}{4} \in \mathbb{Q}$
d $\sqrt{2} \notin \mathbb{Q}$
e $-\frac{1}{4} \notin \mathbb{Q}$
f $2\frac{1}{3} \in \mathbb{Z}$
g $0.3684 \in \mathbb{R}$
h $\dfrac{1}{0.1} \in \mathbb{Z}$

3 Determine whether each of the following numbers is rational, irrational, or neither:

a 8
b -8
c $2\frac{1}{3}$
d $-3\frac{1}{4}$
e $\sqrt{3}$
f $\sqrt{-3}$
g $\sqrt{400}$
h 9.176
i $\frac{1}{0}$
j $\pi - \pi$

4 For each of the following sets:

- **i** list the elements of the set
- **ii** determine whether the set is finite or infinite
- **iii** if the set is finite, find the number of elements in the set.

a $A = \{\text{factors of } 6\}$ **b** $B = \{\text{multiples of } 6\}$ **c** $C = \{\text{factors of } 17\}$

d $D = \{\text{multiples of } 17\}$ **e** $E = \{\text{prime numbers less than } 20\}$

f $F = \{\text{composite numbers between } 10 \text{ and } 30\}$

5 Show that each of the following numbers is rational:

a $0.\overline{7}$ **b** $0.\overline{41}$ **c** $0.\overline{324}$

6 Explain why 0.527 is a rational number.

7 Explain why $0.\overline{9} \in \mathbb{Z}$.

8 Give examples to show that these statements are false:

a The sum of two irrationals is irrational.

b The product of two irrationals is irrational.

EXERCISE 2A

1 **a** $8 \in P$ **b** $k \notin S$ **c** $14 \notin \{\text{odd numbers}\}$
d $n(Y) = 9$

2 **a** true **b** true **c** true **d** true
e false **f** false **g** true **h** true

3 **a** rational **b** rational **c** rational **d** ration
e irrational **f** neither **g** rational **h** ration
i neither **j** rational

4 **a** **i** $A = \{1, 2, 3, 6\}$ **ii** finite **iii** $n(A) = 4$
b **i** $B = \{6, 12, 18, 24, ...\}$ **ii** infinite
c **i** $C = \{1, 17\}$ **ii** finite **iii** $n(C) = 2$
d **i** $D = \{17, 34, 51, 68, ...\}$ **ii** infinite
e **i** $E = \{2, 3, 5, 7, 11, 13, 17, 19\}$ **ii** finite
iii $n(E) = 8$
f **i** $F = \{12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28\}$
ii finite **iii** $n(F) = 13$

5 **a** $0.\overline{7} = \frac{7}{9}$ **b** $0.\overline{41} = \frac{41}{99}$ **c** $0.\overline{324} = \frac{12}{37}$

6 0.527 can be written as $\frac{527}{1000}$, where 527 and 1000 are integers

7 $0.\overline{9} = 1$ and $1 \in \mathbb{Z}$

8 **Note:** There may be other answers.

a $\sqrt{2}, -\sqrt{2}$ are irrational, but $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational.

b $\sqrt{2}, \sqrt{50}$ are irrational, but $\sqrt{2} \times \sqrt{50} = \sqrt{100} = 10$ which is rational.